<u>Continuous Function:</u>

A continuous function is a function whose graph is continuous without any breaks or jumps. i.e., if we are able to draw the graph of a function without even lifting the pencil, then we say that the function is continuous.

Let us study more about the continuity of a function .

What is Continuous Function?

The mathematical definition of the continuity of a function is as follows.

A function f(x) is continuous at a point x = a if

- f(a) exists;
- $\lim_{x \to a} f(x)$ exists;
  [i.e., $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$] and
- Both of the above values are equal. i.e., $\lim_{x \to a} f(x) = f(a)$.

Properties of Continuity:

If two functions f(x) and g(x) are continuous at x = a then

- f + g, f - g, and fg are continuous at x = a.
- f/g is also continuous at x = a provided g(a) ≠ 0.
- If f is continuous at g(a), then the composition function (f o g) is also continuous at x = a.
- All polynomial functions are continuous over the set of all real numbers.
- The absolute value function |x| is continuous over the set of all real numbers.
- Exponential functions are continuous at all real numbers.
- The functions sin x and cos x are continuous at all real numbers.
- The functions tan x, cosec x, sec x, and cot x are continuous on their respective domains.
- The functions like log x, ln x, √x, etc are continuous on their respective domains.

Theorems on Continuous Function:

- **Theorem 1:** All polynomial functions are continuous on (-∞, ∞).
- **Theorem 2:** The functions e^x, sin x, cos x, and arctan x are continuous on (-∞, ∞).

- **Theorem 3:** If two functions f and g are continuous on an interval [a, b], then the algebra of functions: f+g, f-g, and fg are continuous on [a, b]. But f/g is continuous on [a, b] given that f/g is NOT zero anywhere in the interval.
- **Theorem 4:** A rational function is continuous except at the vertical asymptotes.

NOT Continuous Function:

A function that is NOT continuous is said to be a discontinuous function. i.e., the graph of a discontinuous function breaks or jumps somewhere. There are different types of discontinuities as explained below. By the definition of the continuity of a function, a function is NOT continuous in one of the following cases. We can see all the types of discontinuities in the figure below

Jump Discontinuity:

$\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ exist but they are NOT equal. It is called "jump discontinuity" (or) "non-removable discontinuity".

Removable Discontinuity:

$\lim_{x \to a} f(x)$ exists (i.e., $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$) but it is NOT equal to f(a). It is called "removable discontinuity".

Infinite Discontinuity:

The values of one or both of the limits $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ is $\pm \infty$. It is called "infinite discontinuity".

Notes :

- A function is continuous at x = a if and only if $\lim_{x \to a} f(x) = f(a)$.
- It means, for a function to have continuity at a point, it shouldn't be broken at that point.
- For a function to be differentiable, it has to be continuous.
- All polynomials are continuous.
- The functions are NOT continuous at vertical asymptotes.
- The functions are NOT continuous at holes.