③ **Forced oscillation!** - If any body is in oscillatory motion and restoring force resistive force and an external periodic force all act then the oscillations of body are said to be forced oscillation.

(i) The restoring force acting on body is given by $-\mu y$ the resistive force is given as $= -r\frac{dy}{dt}$ and the periodic force is given as $= f_0 \sin pt$ & p is its angular frequency.

Now Net force on the body will be

Net force = Restoring force + Resistive force + periodic force

$$m\frac{d^2y}{dt^2} = -\mu y - r\frac{dy}{dt} + f_0 \sin pt$$

$$m\frac{d^2y}{dt^2} + r\frac{dy}{dt} + \mu y = f_0 \sin pt$$

Divid by m on both side we get

$$\boxed{\frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{\mu}{m}y = f_0 \sin pt} \quad \text{— (I)}$$

put $\frac{r}{m} = 2k$

$\frac{\mu}{m} = \omega^2$ & $\frac{f_0}{m} = F_0$

then eqn (I) reduces as

$$\boxed{\frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + \omega^2 y = F_0 \sin pt} \quad \text{— (II)}$$

After this step, the same steps followed in damped oscillation.

This eqn (II) is II^{nd} order diffn eqn whose soln can be given as

$$y = C.F + P.I \quad \text{— (A)}$$

Now for C.F

$$\frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + \omega^2 y = 0$$

hence C.F will be

$$y = R e^{-kt} \sin(\alpha t + \phi) \quad \text{--- (B)}$$

Now P.I of this eq^n (II) is

$$\boxed{P.I. = A_0 \sin(pt - \delta)} \quad \text{--- (C)}$$

If this eq^n (C) is P.I of eq^n (II) then it must satisfy it so

$$\frac{dy}{dt} = A_0 p \cos(pt - \delta)$$

$$\frac{d^2y}{dt^2} = -A_0 p^2 \sin(pt - \delta)$$

So putting these values in eq^n (II) we get

$$\frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + \omega^2 y = F_0 \sin pt$$

$$\Rightarrow \quad -A_0 p^2 \sin(pt - \delta) + 2kA_0 p \cos(pt - \delta) + \omega^2 A_0 \sin(pt - \delta) = F_0 \sin[(pt - \delta) + \delta]$$

$$-A_0 p^2 \sin(pt - \delta) + 2k A_0 p \cos(pt - \delta) + \omega^2 A_0 \sin(pt - \delta) = F_0 [\sin(pt - \delta)\cos\delta + \cos(pt - \delta)\sin\delta] \quad \text{--- (II)}$$

Now comparing the co-eff' of $\sin(pt - \delta)$ & $\cos(pt - \delta)$

$$(\omega^2 - p^2) A_0 = F_0 \cos\delta \quad \text{--- (IV)}$$

$$2k A_0 p = F_0 \sin\delta \quad \text{--- (V)}$$

Now squaring & Adding eq^n (IV) & (V) we get

$$F_0^2 (\sin^2\delta + \cos^2\delta) = A_0^2 [(\omega^2 - p^2)^2 + 4p^2 k^2]$$

$$A_0 = \frac{F_0}{[(\omega^2 - p^2)^2 + 4k^2 p^2]^{1/2}} \quad \text{--- (D)}$$

So complete solⁿ of diffⁿ eqⁿ (1) will be

$$y = C.F + P.I$$

$$\boxed{y = Re^{-kt}\sin(\alpha t + \beta) + A_0 \sin(pt - \delta)} \quad \text{—— (4)}$$

So, this eqⁿ (4) gives the complete solⁿ of forced oscillation which consist of 2 terms here due to first term the vibrations die out after some time and then IIⁿᵈ terms comes into play due to external force which maintains the oscillation of body and such oscillations are named as forced oscillations.

Sharpness of Resonance :-

In forced oscillations, the amplitude of oscillation is given as

$$\boxed{A_0 = \frac{F_0}{\sqrt{(\omega^2 - p^2)^2 + 4k^2p^2}}} \quad \text{—— (I)}$$

When $\omega = p$ i.e. when the angular freq. of body becomes equal to angular freq. of applied force then Resonance occurs and the amp. of oscillation of body becomes large and can be given as

$$A_0 = \frac{F_0}{\sqrt{4k^2p^2}}$$

$$\boxed{A_0 = \frac{F_0}{2kp}} \quad \text{—— (II)}$$

Now, amplitude A_0 depends upon the damping coefficient k

if $k \to 0 \Rightarrow A_0 \to \infty$